

On the Spectrum of Direct Gaugino Mediation

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ABSTRACT: In direct gauge mediation, the gaugino masses are anomalously small, giving rise to a split SUSY spectrum. Here we investigate the superpartner spectrum in a minimal version of “direct *gaugino* mediation.” We find that the sfermion masses are comparable to those of the gauginos – even in the hybrid gaugino-gauge mediation regime – if the messenger scale is sufficiently small.

KEYWORDS: Supersymmetry Phenomenology

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1 Introduction

If supersymmetry (SUSY) provides an explanation to the hierarchy problem, it should be broken dynamically. A class of such models – “direct gauge mediation” – is obtained by embedding the Minimal Supersymmetric Standard Model (MSSM) group in the flavor symmetry of (deformed) SQCD. However, in such theories, the gaugino masses generically vanish to leading order in SUSY breaking [1]. Consequently, the sfermions are very heavy,¹ and significant fine tuning of the Higgs mass is required.

The main purpose of this note is to investigate models of SUSY breaking and its mediation to the MSSM, which have a simple, generic dynamical origin, but nevertheless lead to a sufficiently degenerate superpartner spectrum. Models of “direct gaugino mediation” [3] provide such a class. These models have a simple low-energy effective description, which allows perturbative computations.

In this work, we compute the soft masses in the minimal version of direct gaugino mediation. The setting of the model is presented in section 2. It is a simple generalization of “minimal gaugino-gauge mediation” [4, 5], whose sparticle spectrum was

¹For a recent study of these split gauge mediated models see [2].

studied in detail in [6]; the messenger sector is a more general one. For completeness, in section 3, we compute the soft masses for a general messenger sector. Then, in section 4, we restrict to the particular subclass of models providing the effective theories of “minimal direct gaugino mediation” [3], and present the soft masses in this class. In section 5, we evaluate the sparticle mass spectrum at the weak scale.

We find that for low scale mediation – when the effective SUSY-breaking scale is comparable to the messenger scale – the gaugino masses can be sufficiently large relative to the scalar masses, even when the mass of the additional gauge particles is also comparable to the messenger scale. We also show that when the extra massive gauge particles are much lighter than the messenger scale, one may ameliorate the little hierarchy problem, allowing a relatively light stop and a heavy Higgs field. Finally, we discuss our results in section 6.

2 Setup

The setting [4, 5], which is a deconstructed version of the extra dimensional theory considered in [7, 8], is summarized in figure 1. It consists of a visible sector containing the MSSM matter fields Q, \tilde{Q} which are all charged under the gauge group \mathcal{G}_A ; a hidden sector containing messenger fields T_i, \tilde{T}_j charged under a different gauge group \mathcal{G}_B and a pair of link fields L, \tilde{L} charged under both gauge groups. Higgsing of the link fields breaks the symmetry $\mathcal{G}_A \times \mathcal{G}_B$ to the diagonal \mathcal{G}_{SM} , which is identified with the MSSM gauge group. The messenger fields are coupled by a superpotential to an F-term SUSY-breaking spurion S , whose θ^2 component attains a SUSY-breaking VEV.

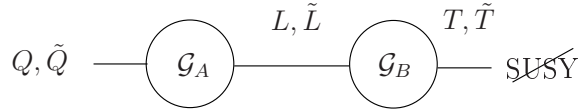


Figure 1. Quiver diagram for the model.

The group \mathcal{G}_A consists of $SU(3) \times SU(2) \times U(1)$ (which we shall often think of as a subgroup of $SU(5)$), while we take \mathcal{G}_B to be $SU(5)$.² The link fields (L, \tilde{L}) are chosen in the $(\mathbf{5}, \mathbf{\bar{5}})$ and $(\mathbf{\bar{5}}, \mathbf{5})$ representations, respectively. An extra field A , which is an adjoint of \mathcal{G}_B , as well as an extra singlet K should be added to give masses to all the link field components. The superpotential reads:

$$\mathcal{W}_{\text{link}} = \kappa_1 \text{Tr}(LA\tilde{L}) + \kappa_2 K \left(\frac{\text{Tr}(L\tilde{L})}{5} - v^2 \right). \quad (2.1)$$

²This case allows perturbative unification in a large subclass of the general models presented in section 3, though it is hard to achieve unification in models that have simple, dynamical realizations. In section 5.2.2 we shall also discuss the case $\mathcal{G}_B = SU(3) \times SU(2) \times U(1)$.

The link fields obtain a VEV proportional to the 5×5 identity matrix:

$$\langle L \rangle = \langle \tilde{L} \rangle = v \mathbf{1}_5. \quad (2.2)$$

The MSSM gauge couplings g_k are related to the gauge couplings of the unbroken theory (i.e. before spontaneous symmetry breaking) as follows:

$$\frac{1}{g_k^2} = \frac{1}{g_{A_k}^2} + \frac{1}{g_B^2}, \quad (2.3)$$

where $k = 1, 2, 3$ corresponds respectively to the $U(1)$, $SU(2)$ and $SU(3)$ gauge group, while the coupling g_B is $SU(5)$ invariant (i.e. the couplings of the $SU(3)$, $SU(2)$ and $U(1)$ subgroups are all identical). A linear combination of the A, B gauge multiplets gets a mass due to the super Higgs mechanism:

$$m_{v_k}^2 = 2v^2(g_{A_k}^2 + g_B^2). \quad (2.4)$$

The imaginary part of $l_k = (L_k - \tilde{L}_k)/\sqrt{2}$ is eaten via the Higgs mechanism; the real part of l_k corresponds to the scalar in the massive vector multiplet with mass m_{v_k} .

In [6], the sfermion soft masses have been computed at two loops in the case of a minimal messenger sector; in this note we will extend the computation to the case of a more generic weakly coupled messenger sector, which we will discuss in detail in the next section, see eq. (3.1). A specific example motivated by the dynamical realization proposed in [3] is further studied in more detail.

The two-loop calculation is not a good approximation when the VEV v is much smaller than the messenger scale Ω . In this limit, $v \ll \Omega$, the two-loop sfermion mass is negligible; the leading contribution then comes from three loops. An approximate computation of these contributions was performed in [9]. For the concrete example discussed in section 4, we will compare the spectrum in the hybrid regime (i.e. v of the order of Ω) to the spectrum in the limit $v \ll \Omega$.

3 Soft masses

Let us consider the following weakly-coupled sector of N messenger pairs coupled to a SUSY-breaking F-term spurion S and a D-term spurion V [10, 11]:

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left(T_i^\dagger (\delta_{ij} + V \tilde{\lambda}_{ij}) T_j + \tilde{T}_i^\dagger (\delta_{ij} + V \tilde{\lambda}_{ij}) \tilde{T}_j \right) \\ & + \int d^2\theta \tilde{T}_i (S \lambda_{ij} + m_{ij}) T_j + \text{c.c.}, \end{aligned} \quad (3.1)$$

where $i, j = 1, \dots, N$. We use the basis in which the matrix m is diagonal with real eigenvalues m_i ; with this choice, by requiring messenger parity and CP conservation, the matrix λ is real and symmetric. The matrix $\tilde{\lambda}$ must be Hermitian. In order

to avoid a non-zero messenger supertrace (and hence loosing calculability [12]), we require $\text{Tr } \tilde{\lambda} = 0$. The F and D-term spurions acquire the expectation values

$$S|_{\theta^2} = f, \quad V|_{\theta^4} = D, \quad (3.2)$$

which we take to be real. The fermions in the messenger sector $(\psi_{T_i}, \psi_{\tilde{T}_i})$ have Dirac masses m_i while the complex scalars (T_i, \tilde{T}_i^*) have the following mass-squared matrix

$$\mathcal{M} = \begin{pmatrix} m^2 - D\tilde{\lambda} & -f\lambda \\ -f\lambda & m^2 - D\tilde{\lambda} \end{pmatrix}. \quad (3.3)$$

In the basis (T_{-i}, T_{+i}) , where $T_{\pm i} \equiv (T_i \pm \tilde{T}_i^*)/\sqrt{2}$, the matrix \mathcal{M} reads:

$$\begin{pmatrix} m^2 + f\lambda - D\tilde{\lambda} & 0 \\ 0 & m^2 - f\lambda - D\tilde{\lambda} \end{pmatrix} \equiv \begin{pmatrix} \mathcal{M}_+ & 0 \\ 0 & \mathcal{M}_- \end{pmatrix}. \quad (3.4)$$

Let us denote by U_{\pm} the unitary matrices which diagonalize \mathcal{M}_{\pm} :

$$U_{\pm}^{\dagger} \mathcal{M}_{\pm} U_{\pm} = \text{diag}(m_{\pm 1}^2, \dots, m_{\pm N}^2). \quad (3.5)$$

The one-loop gaugino masses due to gauge mediation are [10, 11]:

$$M_{\tilde{g}_k} = n_k \frac{\alpha_k}{4\pi} \tilde{G}, \quad \tilde{G} = \sum_{\pm, i, j} \mp (U_{\pm}^{\dagger})_{ij} (U_{\pm})_{ji} m_j \frac{m_{\pm i}^2 \log \frac{m_{\pm i}^2}{m_j^2}}{m_{\pm i}^2 - m_j^2}, \quad (3.6)$$

where $\alpha_k = g_k^2/(4\pi)$, $k = 1, 2, 3$ labels the gauge groups $U(1), SU(2), SU(3)$, respectively, and n_k is the Dynkin index of a *single* messenger pair with respect to the corresponding gauge group. In the case of the gaugino-gauge mediation setting discussed in section 2, g_k is the effective low-energy gauge coupling defined in eq. (2.3), which corresponds to the unbroken combination of the groups \mathcal{G}_{A_k} and \mathcal{G}_B .

Let us now review the computation of the sfermion masses in gauge mediation [11] in the case of an $SU(n)$ gauge group. It is convenient to use the global $SU(n)$ current multiplet formalism of [13]. The symmetry current j_{μ}^a is embedded in a real superfield \mathcal{J}^a , which also contains a scalar J^a and a spinor j_{α}^a , where $a = 1, \dots, n^2 - 1$ is the adjoint index. The functions $C_i(x)$ parametrize the correlators as follows:

$$\begin{aligned} \langle J^a(x) J^b(0) \rangle &\equiv C_0(x) \delta^{ab}, \\ \langle j_{\alpha}^a(x) j_{\dot{\beta}}^{*b}(0) \rangle &\equiv -i \sigma_{\alpha\dot{\beta}}^{\mu} \partial_{\mu} C_{1/2}(x) \delta^{ab}, \\ \langle j_{\mu}^a(x) j_{\nu}^b(0) \rangle &\equiv (\eta_{\mu\nu} \partial^2 - \partial_{\mu} \partial_{\nu}) C_1(x) \delta^{ab}. \end{aligned} \quad (3.7)$$

In the weakly coupled setting that we consider, the components of \mathcal{J}^a can be written explicitly [10, 14]:

$$\begin{aligned} J^a &= T_i^{*} t^a T_i - \tilde{T}_i^{*} t^a \tilde{T}_i, \\ j_{\alpha}^a &= -\sqrt{2}i \left(T_i^{*} t^a \psi_{T_i\alpha} - \tilde{T}_i^{*} t^a \psi_{\tilde{T}_i\alpha} \right), \\ j_{\mu}^a &= i \left(T_i^{*} t^a \partial_{\mu} T_i - T_i^{*} t^a \partial_{\mu} T_i - \tilde{T}_i^{*} t^a \partial_{\mu} \tilde{T}_i + \tilde{T}_i^{*} t^a \partial_{\mu} \tilde{T}_i \right) + \psi_{T_i} \sigma_{\mu} t^a \psi_{T_i}^{*} - \psi_{\tilde{T}_i} \sigma_{\mu} t^a \psi_{\tilde{T}_i}^{*}, \end{aligned} \quad (3.8)$$

where t^a are the generators of $SU(n)$. In momentum space, the correlators are:

$$\begin{aligned}
\tilde{C}_0(p) &= \sum_{\pm, i, j} \left(U_{\pm}^{\dagger} U_{\mp} \right)_{ij} \left(U_{\mp}^{\dagger} U_{\pm} \right)_{ji} I(p, m_{\pm i}, m_{\mp j}), \\
\tilde{C}_{1/2}(p) &= \frac{1}{p^2} \sum_{\pm, i} (J(m_{\pm i}) - J(m_i)) \\
&\quad + \frac{1}{p^2} \sum_{\pm, i, j} (U_{\pm}^{\dagger})_{ij} (U_{\pm})_{ji} (p^2 + m_{\pm i}^2 - m_j^2) I(p, m_{\pm i}, m_j), \\
\tilde{C}_1(p) &= \frac{1}{3p^2} \sum_{\pm, i} ((p^2 + 4m_{\pm i}^2) I(p, m_{\pm i}, m_{\pm i}) + 4J(m_{\pm i})) \\
&\quad + \frac{4}{3p^2} \sum_i ((p^2 - 2m_i^2) I(p, m_i, m_i) - 2J(m_i)), \tag{3.9}
\end{aligned}$$

where

$$I(p, m_1, m_2) \equiv \int \frac{d^4 q}{(2\pi)^4} \frac{1}{((p+q)^2 + m_1^2)(q^2 + m_2^2)}, \quad J(m) \equiv \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 + m^2)}.$$

The sfermion soft masses are then given by:

$$m_{\tilde{f}}^2 = -8\pi^2 \sum_{k=1}^3 \alpha_k^2 C_{\tilde{f}, k} n_k \int \frac{d^4 p}{(2\pi)^4} \frac{f_k(p^2)}{p^2} \left(\tilde{C}_0(p) - 4\tilde{C}_{1/2}(p) + 3\tilde{C}_1(p) \right), \tag{3.10}$$

where the index k sums over the gauge groups $U(1)$, $SU(2)$ and $SU(3)$; $C_{\tilde{f}, k}$ is the quadratic Casimir of the sfermion \tilde{f} and n_k is the Dynkin index of a *single* pair of messengers. Finally, the function $f_k(p^2)$ depends on the gauge group as well as the specific model under consideration. In the case of gauge mediation it is simply $f_k(p^2) = 1$ for all k . A direct calculation [11] using the two-loop integrals in [15] and setting $f_k(p^2) = 1$ gives rise to:

$$\begin{aligned}
m_{\tilde{f}}^2 &= 2 \sum_{k=1}^3 \left(\frac{\alpha_k}{4\pi} \right)^2 C_{\tilde{f}, k} n_k \left\{ \sum_{\pm, i} (m_{\pm i}^2 \log m_{\pm i}^2 - m_i^2 \log m_i^2) \right. \\
&\quad + \frac{1}{2} \sum_{\pm, i, j} (U_{\pm}^{\dagger} U_{\mp})_{ij} (U_{\mp}^{\dagger} U_{\pm})_{ji} m_{\pm i}^2 \text{Li}_2 \left(1 - \frac{m_{\mp j}^2}{m_{\pm i}^2} \right) \\
&\quad \left. - 2 \sum_{\pm, i, j} (U_{\pm}^{\dagger})_{ij} (U_{\pm})_{ji} m_{\pm i}^2 \text{Li}_2 \left(1 - \frac{m_j^2}{m_{\pm i}^2} \right) \right\}, \tag{3.11}
\end{aligned}$$

where the dilogarithm is defined by $\text{Li}_2(x) = -\int_0^1 \frac{dt}{t} \log(1 - xt)$.

In order to compute the two-loop sfermion masses in the gaugino-gauge mediation setting discussed in section 2, we should use the momentum dependent function found in [6, 16, 17]:

$$f_k(p^2) = \left(\frac{m_{v_k}^2}{p^2 + m_{v_k}^2} \right)^2. \tag{3.12}$$

Using the two-loop integrals in [15, 18], an explicit calculation gives:

$$m_{\tilde{f}}^2 = 2 \sum_{k=1,2,3} \left(\frac{\alpha_k}{4\pi} \right)^2 C_{\tilde{f},k} n_k \tilde{S}_k, \quad (3.13)$$

$$\tilde{S}_k = \sum_{\pm, i, j} (U_{\pm}^{\dagger} U_{\mp})_{ij} (U_{\mp}^{\dagger} U_{\pm})_{ji} a_{\pm ij}^k + \sum_{\pm, i, j} (U_{\pm}^{\dagger})_{ij} (U_{\pm})_{ji} b_{\pm ij}^k + \sum_{\pm, i} c_{\pm i}^k, \quad (3.14)$$

where we have defined

$$\begin{aligned} a_{\pm ij}^k &= \frac{1}{2} m_{\pm i}^2 \left(\text{Li}_2 \left(1 - \frac{m_{\mp j}^2}{m_{\pm i}^2} \right) - h \left(\frac{m_{v_k}^2}{m_{\pm i}^2}, \frac{m_{\mp j}^2}{m_{\pm i}^2} \right) \right), \\ b_{\pm ij}^k &= \left(\frac{2(m_j^2 - m_{\pm i}^2)}{m_{v_k}^2} + 1 \right) \left(m_j^2 h \left(\frac{m_{v_k}^2}{m_j^2}, \frac{m_{\pm i}^2}{m_j^2} \right) + m_{\pm i}^2 h \left(\frac{m_{v_k}^2}{m_{\pm i}^2}, \frac{m_j^2}{m_{\pm i}^2} \right) \right) \\ &\quad + (m_j^2 - m_{\pm i}^2) \left(\frac{m_j^2}{m_{v_k}^2} \log^2 \left(\frac{m_j^2}{m_{\pm i}^2} \right) + h \left(\frac{m_{\pm i}^2}{m_{v_k}^2}, \frac{m_j^2}{m_{v_k}^2} \right) \right) \\ &\quad + 2 \left(\frac{(m_j^2 - m_{\pm i}^2)^2}{m_{v_k}^2} - m_{\pm i}^2 \right) \text{Li}_2 \left(1 - \frac{m_j^2}{m_{\pm i}^2} \right), \\ c_{\pm i}^k &= m_{\pm i}^2 \left(-\frac{2m_{\pm i}^2}{m_{v_k}^2} + \log m_{\pm i}^2 + h \left(\frac{m_{\pm i}^2}{m_{v_k}^2}, \frac{m_{\pm i}^2}{m_{v_k}^2} \right) + \left(\frac{4m_{\pm i}^2}{m_{v_k}^2} - \frac{1}{2} \right) h \left(\frac{m_{v_k}^2}{m_{\pm i}^2}, 1 \right) \right) \\ &\quad - m_i^2 \left(-\frac{2m_i^2}{m_{v_k}^2} + \log m_i^2 + h \left(\frac{m_i^2}{m_{v_k}^2}, \frac{m_i^2}{m_{v_k}^2} \right) + \left(\frac{4m_i^2}{m_{v_k}^2} + 1 \right) h \left(\frac{m_{v_k}^2}{m_i^2}, 1 \right) \right). \end{aligned} \quad (3.15)$$

This expression generalizes the one found in [6] for a minimal messenger sector. The function h is defined by the integral:

$$h(a, b) = \int_0^1 dx \left(1 + \text{Li}_2(1 - \mu^2) - \frac{\mu^2}{1 - \mu^2} \log \mu^2 \right), \quad \mu^2 = \frac{ax + b(1 - x)}{x(1 - x)};$$

an analytical expression for h can be found in [18]. Note that eq. (3.11) is recovered from eq. (3.13) by taking the limit $m_{v_k} \rightarrow \infty$.

4 A dynamical realization

4.1 Description of the model

A realization of the gaugino mediation setup described in section 2 in massive SQCD with singlets was studied in [3]. Consider $SU(N_c)$ SQCD with N_f quark multiplets Q in the fundamental representation as well as N_f anti-fundamental multiplets \tilde{Q} , which are labeled by the indices $i, j = 1, \dots, n$ and $a, b = n + 1, \dots, N_f$, where $n = N_f - N_c$. The singlets H_i^a and \tilde{H}_a^i are introduced as well. The superpotential reads

$$\mathcal{W}_e = (Q^i \ Q^a) \begin{pmatrix} m_{(1)} \delta_i^j & H_i^b \\ \tilde{H}_a^j & m_{(2)} \delta_a^b \end{pmatrix} \begin{pmatrix} \tilde{Q}_j \\ \tilde{Q}_b \end{pmatrix}. \quad (4.1)$$

The masses of the two subsets of quarks, $m_{(1)}$ and $m_{(2)}$, are both taken to be much smaller than the confinement scale Λ . For $N_f < 3N_c$ this theory is asymptotically free; the Seiberg dual [19], for $N_f > N_c + 1$, is given in terms of an $SU(n)$ gauge theory. We focus on the regime $N_f < 3N_c/2$, where the dual theory is IR-free. The superpotential of the dual theory is then:

$$\mathcal{W}_m = qM\tilde{q} + \Lambda H_i^a M_a^i + \Lambda \tilde{H}_a^i M_i^a + \Lambda \text{Tr}(\mathbf{m} M) , \quad (4.2)$$

where q, \tilde{q} are the N_f dual quarks, M is the meson field and $\mathbf{m} = m_{(1)}\delta_i^j + m_{(2)}\delta_a^b$.

After integrating out the heavy states, the meson and the dual quarks can be decomposed into the following blocks (in flavor space, while the color indices for the dual quarks are implicit):

$$M = \left(\begin{array}{c|cc} N_{n \times n} & & \\ \hline & X_{n \times n} & Y_{n \times p} \\ \hline & \tilde{Y}_{p \times n} & Z_{p \times p} \end{array} \right) , \quad q = (\chi_n | \eta_n | \rho_p) , \quad \tilde{q} = \left(\begin{array}{c} \tilde{\chi}_n \\ \tilde{\eta}_n \\ \tilde{\rho}_p \end{array} \right) , \quad (4.3)$$

where the subscripts denote the dimension of each block and $p = 2N_c - N_f$. In order to cancel as many F-terms as possible, the VEVs are chosen as follows:

$$\chi = \tilde{\chi} = \sqrt{\Lambda m_{(1)}} \mathbf{1}_n , \quad \eta \tilde{\eta} = \Lambda m_{(2)} \mathbf{1}_n , \quad (4.4)$$

while the Z components are pseudo-moduli and the other VEVs are equal to zero. The one-loop Coleman-Weinberg potential sets $Z = 0$ and $\eta = \tilde{\eta}$; the SUSY-breaking sector is similar to the ISS model [20].

The fields $\chi, \tilde{\chi}$ are identified with the link fields L, \tilde{L} of the quiver diagram in figure 1 and the $SU(n)_\chi$ flavor group is identified with the gauge group \mathcal{G}_A . The VEV of η breaks the $SU(n)_\eta$ global flavor group and the dual $SU(n)$ gauge group to a diagonal combination, which is identified with the gauge group \mathcal{G}_B . Finally, the field Z is the SUSY-breaking spurion S while

$$T = (T_1, T_2) = (\rho, Y) , \quad \tilde{T} = (\tilde{T}_1, \tilde{T}_2) = (\tilde{\rho}, \tilde{Y}) , \quad (4.5)$$

are the messengers.

The model has an accidental R-symmetry which is not broken by the metastable vacuum; for this reason the gaugino masses are zero. This accidental symmetry should thus be broken, e.g. by adding a quartic deformation in the UV, as in [21]. This deformation turns in the IR into a superpotential term $\delta\mathcal{W} \propto \text{Tr} Z^2$, giving Z a non-zero VEV which we choose to be $\langle Z \rangle = \omega \mathbf{1}_p$.

4.2 Soft masses

In this section we apply the generic results of section 3 to the model briefly described in section 4.1. There is a total of $2p$ messengers which can be organized as p copies of

the two messenger pairs (T_1, \tilde{T}_1) and (T_2, \tilde{T}_2) . The coupling between the messengers and the spurion S is given by eq. (3.1) with

$$m = \begin{pmatrix} \omega & \Omega \\ \Omega & 0 \end{pmatrix} \otimes \mathbf{1}_p, \quad \lambda = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \mathbf{1}_p, \quad (4.6)$$

where $\Omega = \sqrt{\Lambda m_{(2)}}$ and ω is the VEV of each diagonal element of Z . The VEV of the link field L is $v = \sqrt{\Lambda m_{(1)}}$. In order to satisfy the relation $N_f < 3N_c/2$ for $n = 5$ (i.e. corresponding to an $SU(5)$ GUT), we have to require $p \geq 6$; the values of $(n, p) = (5, 6)$ correspond to $N_c = 11$, $N_f = 16$. A more generic expression is:

$$N_c = n + p, \quad N_f = 2n + p. \quad (4.7)$$

In the following we will first consider the formal case $p = 1$ which corresponds to two messengers. The only effect of general p is an overall p -factor in the soft masses, which we shall reintroduce in the end.

It is useful to pass to the basis where the masses of the fermionic messengers are diagonal:

$$m = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad \lambda = \frac{1}{2} \mathbf{1}_2 + \frac{1}{2\sqrt{4\Omega^2 + \omega^2}} \begin{pmatrix} -\omega & 2\Omega \\ 2\Omega & \omega \end{pmatrix}, \quad (4.8)$$

where

$$m_{1,2} = \frac{1}{2}(\omega \mp \sqrt{4\Omega^2 + \omega^2}). \quad (4.9)$$

The squared masses of the bosonic messengers are:

$$m_{\pm s}^2 = \frac{1}{2} \left(\pm f + 2\Omega^2 + \omega^2 + (-1)^s \sqrt{f^2 + 2(2\Omega^2 \pm f)\omega^2 + \omega^4} \right), \quad (4.10)$$

where $s = 1, 2$. The diagonalization matrices for the bosonic messenger masses in eq. (3.5) are given by:

$$U_{\pm} = \begin{pmatrix} \cos \theta_{\pm} & \sin \theta_{\pm} \\ -\sin \theta_{\pm} & \cos \theta_{\pm} \end{pmatrix}, \quad (4.11)$$

where the rotation angles are:

$$\tan \theta_{\pm} = \frac{\omega(\mp f - 4\Omega^2 - \omega^2) + \sqrt{(4\Omega^2 + \omega^2)(f^2 \pm 2f + 4\Omega^2\omega^2 + \omega^4)}}{\pm 2f\Omega}. \quad (4.12)$$

The following variables are introduced for convenience:

$$x = \frac{f}{\Omega^2}, \quad y_k = \frac{m_{v_k}}{\Omega}, \quad z = \frac{\omega}{\Omega}. \quad (4.13)$$

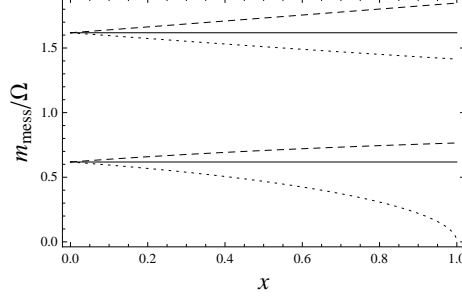


Figure 2. Messenger masses as functions of x in units of Ω for $z = 1$.

In the numerical examples that we will consider, the relative difference between the parameters y_k , $k = 1, 2, 3$, is very small; hence, we will often simply denote by y their average value. Analogously, we will denote by m_v the average of m_{v_k} . A plot of the messenger masses as a function of x for $z = 1$ is shown in figure 2.

Using eq. (3.6), we can compute the gaugino soft masses:

$$M_{\tilde{g}_k} = p \frac{\alpha_k}{4\pi} \frac{f}{\Omega} G(x, z), \quad G(x, z) = \frac{\Omega}{f} \tilde{G}, \quad (4.14)$$

where G is the function calculated for $p = 1$ (i.e. corresponding to 2 messengers) and $\alpha_k = g_k^2/(4\pi)$, where g_k is defined in eq. (2.3). The messengers are fixed here in the $\mathbf{5} + \bar{\mathbf{5}}$ representation of $SU(5)$. Notice the reinstated p , which renders the result valid for any $p \geq 1$. A plot of the function G is shown in figure 3; for fixed x , we obtain the highest gaugino masses for $z \approx 1$.

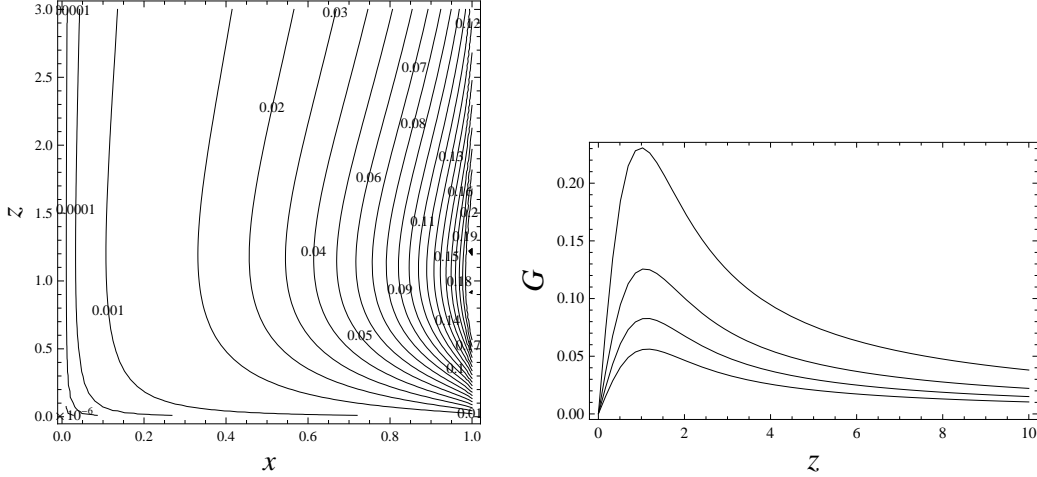


Figure 3. Left panel: The function $G(x, z)$ for the gaugino masses, as defined in eq. (4.14). Right panel: The function $G(x, z)$ as a function of z with $x = 1, 0.9, 0.8, 0.7$ (from top to bottom).

In the limit $x \rightarrow 1$, the function $G(x, z)$ has the following behavior:

$$\lim_{x \rightarrow 1} G(x, \omega/\Omega) \sim \begin{cases} 0.39 \Omega/\omega, & \omega \gg \Omega, \\ 0.23, & \omega \sim \Omega, \\ 0.45 \omega/\Omega, & \omega \ll \Omega. \end{cases} \quad (4.15)$$

Using eq. (3.13), we obtain for the sfermion mass-squared:

$$m_{\tilde{f}}^2 = 2p \sum_k \left(\frac{\alpha_k}{4\pi} \right)^2 C_{\tilde{f},k} \left(\frac{f}{\Omega} \right)^2 S(x, y_k, z), \quad S(x, y_k, z) = \left(\frac{\Omega}{f} \right)^2 \tilde{S}_k. \quad (4.16)$$

The function S is again calculated for the case $p = 1$, while the explicit factor of p appears as the promised multiplicative factor and hence renders the result valid for any $p \geq 1$. The function S is shown in figure 4 in the large y limit as well as for $y = 1$. As already mentioned, in the large y limit, eq. (4.16) reduces to the gauge mediation expression given by eq. (3.11).

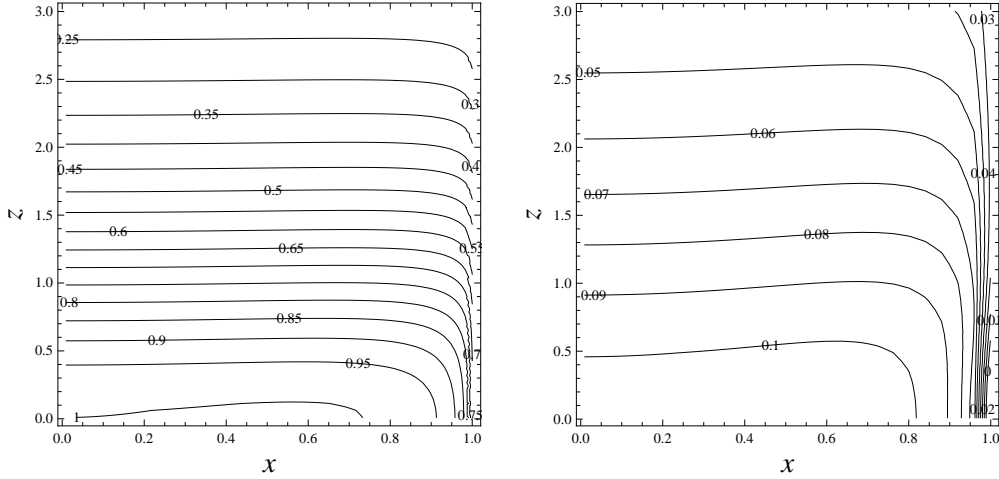


Figure 4. Left panel: The function $S(x, y, z)$ for the sfermion squared masses (as defined in eq. (4.16)), in the limit $y \rightarrow \infty$. Right panel: The function $S(x, 1, z)$, viz. with $y = 1$.

5 Weak scale spectrum

5.1 Hybrid regime

From figure 3, one can see that the one-loop gaugino masses are indeed highly suppressed when x is small. The regime that is more interesting phenomenologically (i.e. in order to avoid large sfermion masses) thus corresponds to relatively large x , i.e. roughly $0.8 < x < 1$, and $z \approx 1$. We are then forced to consider only low messenger scales, such as $10^5 - 10^6$ GeV. In this regime of parameter space, the

various masses of the messengers are rather split (see figure 2), however, the average messenger scale is roughly Ω in any case.

The two-loop sfermion masses computed in the previous section can be trusted only in the regime where the function $S(x, y, z)$ in eq. (4.16) is sufficiently larger than a loop factor $\alpha/(4\pi) \approx 0.01$. In the model under consideration, this is usually the case for $y \gtrsim 1$. Examples of spectra in this regime are shown in table 1 (for $p = 1$) and in table 2 (for $p = 6$). The case of $p = 6$ corresponds to the minimal $N_c = 11$ and $N_f = 16$ embedding in the dynamical model discussed in section 4, while $p = 1$ does not correspond to any known dynamical embedding, but formally it is the minimal case with vanishing gaugino mass at the leading order in SUSY breaking – the case with two messengers. The spectra were obtained using the program SOFTSUSY [22] to solve the renormalization group (RG) equations from the messenger scale Ω to the weak scale. The mass splitting among the messengers is not negligible (e.g. there is a ratio of 6 between the mass of the heaviest and the lightest messengers for $x = 0.8$ and a ratio of 20 for $x = 0.98$), and thus a priori there is no clear-cut scale from which it is most appropriate to start the RG evolution. The imprecision arising from this fact is however of the order of higher order corrections in α_k . We did check though that the results are not that sensitive to this choice in the specific examples that we have considered. The trilinear scalar soft terms were set to zero at the messenger scale due to an extra loop suppression. We assume that only the gauge mediated contribution is present for the soft masses of H_u, H_d at the messenger scale while $\mu, B\mu$ are computed by imposing electroweak symmetry breaking as well as the value of $\tan \beta$.

For $p = 1$ we get a (mostly bino) neutralino NLSP in all of the parameter space, while for $p = 6$ both neutralino and stau NLSPs are possible. The neutralino NLSP is promptly decaying, because $\sqrt{f} \lesssim 10^6 \text{ GeV}$. The experimental constraints from 36 pb^{-1} of LHC data discussed in [23] are relevant in the case of neutralino NLSP, implying that the gluino mass has to be greater than 600 GeV . For most spectra in the tables we have been able to put the Higgs mass near 115 GeV , whereas in a few examples it was necessary to push it a bit in order to obtain a gluino mass above 800 GeV .

5.2 Gaugino mediation regime

5.2.1 $\mathcal{G}_B = SU(5)$

This regime is defined by $m_{v_k} \ll \Omega$ and in this limit the link field potential due to SUSY breaking is no longer negligible compared to the tree-level potential arising from the superpotential in eq. (2.1). For this reason we redefine the scale v in eq. (2.1) to be Λ_v :

$$\mathcal{W}_{\text{link}} = \kappa_1 \text{Tr}(L A \tilde{L}) + \kappa_2 K \left(\frac{\text{Tr}(L \tilde{L})}{5} - \Lambda_v^2 \right), \quad (5.1)$$

Ω x, y, z	1.7×10^6 0.8,1,1	5.15×10^5 0.98,1.8,1
S_1, S_2, S_3	0.08,0.09,0.10	0.13, 0.13, 0.15
$M_{\tilde{g}_1}, M_{\tilde{g}_2}, M_{\tilde{g}_3}$	179,303,601	153,266,555
$m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}$	3974,3769,3748	1911,1821,1812
$m_{\tilde{L}}, m_{\tilde{e}}$	1381,692	631,309
$\mu, B\mu$	1565,2011 ²	784,991 ²
$m_{\tilde{g}}$	941	818
$m_{\tilde{\chi}_0}$	154,308,1545,1546	133,263,774,778
$m_{\tilde{\chi}_{\pm}}$	308,1547	263,779
$m_{\tilde{u}_L}, m_{\tilde{d}_L}$	4013,4014	1961,1963
$m_{\tilde{u}_R}, m_{\tilde{d}_R}$	3802,3781	1870,1862
$m_{\tilde{t}_1}, m_{\tilde{t}_2}$	3408,3815	1688,1872
$m_{\tilde{b}_1}, m_{\tilde{b}_2}$	3742,3814	1841,1870
$m_{\tilde{e}_R}, m_{\tilde{e}_L}, m_{\tilde{\nu}_e}$	700,1378,1376	317,635,630
$m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\tilde{\nu}_\tau}$	679,1374,1370	304,635,628
m_{h_0}	121	117
$m_{H_0}, m_{A_0}, m_{H_{\pm}}$	1965,1965,1967	947,947,950

Table 1. Examples of weak scale spectra in the case of $p = 1$ (two messengers), with $\mu > 0$, $\tan \beta = 20$ and $\alpha_B^{-1} = 4$ at the scale Ω . All the masses are in GeV. y is the average of y_1, y_2, y_3 while S_k is an abbreviation for $S(x, y_k, z)$. The input masses ($M_{\tilde{g}_k}, m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{L}}, m_{\tilde{e}}$) are given at the messenger scale Ω ; $(\mu, B\mu)$ are also evaluated at Ω . The other masses in the table are the MSSM pole masses. Here we have included only the two-loop contribution to the sfermion soft masses.

while we continue to denote by v the VEV of the link field, which is now obtained by minimizing the link field potential:

$$V_{\text{link}} = V_F + V_D + m_L^2 \text{Tr} (LL^\dagger + \tilde{L}^\dagger \tilde{L}), \quad (5.2)$$

where V_F and V_D are the supersymmetric F and D-term potentials and m_L is the gauge mediated soft mass for the link field, which in the limit $m_{v_k} \ll \Omega$ can be computed from the gauge mediation expression (3.11), (3.13), with α_k replaced by α_B ,

$$m_L^2 = 2 \left(\frac{\alpha_B}{4\pi} \right)^2 C_L p \left(\frac{f}{\Omega} \right)^2 S(x, \infty, z), \quad (5.3)$$

where $C_L = 12/5$ is the Casimir of the link field. The heavy vector still has the mass m_{v_k} given by eq. (2.4) while the link field scalar instead gets a mass $m_{s_k}^2 = m_{v_k}^2 + 2m_L^2$.

For $m_{v_k} \ll \Omega$, the two-loop sfermion masses computed in sections 3 and 4 are negligible and hence the leading order sfermion masses are generated at three loops. Thus we can use the method described in [9] to compute the spectrum.

Ω	2.6×10^5	1.65×10^5
x, y, z	0.8,1,1	0.98,1.8,1
S_1, S_2, S_3	0.08,0.09,0.10	0.13, 0.13, 0.16
$M_{\tilde{g}_1}, M_{\tilde{g}_2}, M_{\tilde{g}_3}$	159,278,598	288,505,1094
$m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}$	1616,1545,1538	1537,1469,1463
$m_{\tilde{L}}, m_{\tilde{e}}$	517,251	490,237
$\mu, B\mu$	648,833 ²	611,858 ²
$m_{\tilde{g}}$	849	1419
$m_{\tilde{\chi}_0}$	139,273,640,648	254,478,605,631
$m_{\tilde{\chi}_{\pm}}$	274,648	478,630
$m_{\tilde{u}_L}, m_{\tilde{d}_L}$	1678,1680	1716,1718
$m_{\tilde{u}_R}, m_{\tilde{d}_R}$	1606,1600	1651,1646
$m_{\tilde{t}_1}, m_{\tilde{t}_2}$	1464,1610	1525,1660
$m_{\tilde{b}_1}, m_{\tilde{b}_2}$	1582,1607	1630,1654
$m_{\tilde{e}_R}, m_{\tilde{e}_L}, m_{\tilde{\nu}_e}$	259,524,518	252,514,508
$m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\tilde{\nu}_\tau}$	248,524,516	241,515,506
m_{h_0}	116	116
$m_{H_0}, m_{A_0}, m_{H_{\pm}}$	780,781,785	748,748,753

Table 2. Examples of weak scale spectra in the case of $p = 6$ (six identical copies of two messengers), with $\mu > 0$, $\tan \beta = 20$ and $\alpha_B^{-1} = 4$ at the scale Ω .

Let us denote by m_v the average of the masses m_{v_k} . It is useful to split the renormalization of the soft parameters into two parts, $m_v < \mu < \Omega$ and $\mu < m_v$, according to the RG scale μ . At the scale Ω the sfermion masses are taken to be zero, while the gaugino of the gauge group \mathcal{G}_B gets a soft mass $M_{\tilde{g},B}$, which can be computed from the expression (3.6, 4.14), with α_k replaced by α_B . The link field scalars L, \tilde{L} also get a gauge mediated soft mass m_L . In the present example, $M_{\tilde{g},B}$ vanishes to leading order in SUSY breaking and hence is suppressed compared to m_L .

From the scale Ω to m_v , the renormalization group equations for $(m_L^2, M_{\tilde{g},B})$ are given by [24]:

$$\frac{d m_L^2}{d(\log \mu)} = -\frac{C_L}{2\pi^2} g_B^2 M_{\tilde{g},B}^2, \quad \frac{d M_{\tilde{g},B}}{d(\log \mu)} = \frac{b_B}{8\pi^2} g_B^2 M_{\tilde{g},B}, \quad (5.4)$$

where $C_L = 12/5$ is the Casimir for the link field and $b_B = -5$. At the scale m_v , the following sfermion masses are generated by integrating out the link field and the heavy gaugino [9]:

$$m_{\tilde{f}}^2 = \sum_k \frac{\alpha_k}{4\pi} C_{\tilde{f},k} \left(\frac{2\alpha_k(\alpha_B - 3\alpha_k)}{\alpha_B^2} M_{\tilde{g},B}^2 + \frac{\alpha_k}{\alpha_B - \alpha_k} m_{v_k}^2 \log \left(1 + \frac{2m_L^2}{m_{v_k}^2} \right) \right). \quad (5.5)$$

The (H_u, H_d) soft masses at the scale m_v are:

$$m_{H_{u,d}}^2 = m_{\tilde{l}}^2 + \lambda_{t,b}^2 \frac{\alpha_3}{4\pi} \left(\frac{2\alpha_3^2}{\alpha_B^2} M_{\tilde{g},B}^2 - \frac{\alpha_3}{\alpha_B - \alpha_3} \frac{m_{v_3}^2}{2} \log \left(1 + \frac{2m_L^2}{m_{v_3}^2} \right) \right), \quad (5.6)$$

where $m_{\tilde{l}}$ is the soft mass of the slepton doublet and $\lambda_{t,b}$ are the Yukawa couplings of the top and bottom, respectively. The MSSM gaugino masses are determined from $M_{\tilde{g},B}$:

$$M_{\tilde{g}_k} = \frac{\alpha_k}{\alpha_B} M_{\tilde{g},B}. \quad (5.7)$$

Furthermore, rather small trilinear terms are generated [9] at the scale m_v by integrating out the heavy gaugino:

$$a_t = \frac{\lambda_t}{4\pi} \left(\frac{16}{3} \alpha_3^2 + 3\alpha_2^2 + \frac{13}{15} \alpha_1^2 \right) \frac{M_{\tilde{g},B}}{\alpha_B}, \quad (5.8)$$

$$a_b = \frac{\lambda_b}{4\pi} \left(\frac{16}{3} \alpha_3^2 + 3\alpha_2^2 + \frac{7}{15} \alpha_1^2 \right) \frac{M_{\tilde{g},B}}{\alpha_B}, \quad a_\tau = \frac{\lambda_\tau}{4\pi} \left(3\alpha_2^2 + \frac{9}{5} \alpha_1^2 \right) \frac{M_{\tilde{g},B}}{\alpha_B},$$

where λ_τ is the Yukawa coupling for the τ . Below m_v the RG equations are solved using SOFTSUSY [22]. Examples of spectra are shown in tables 3 and 4. It turns out that these spectra are rather similar to the ones considered in the previous section for $m_v \approx \Omega$.

It is interesting to compare the magnitude of the two-loop sfermion soft masses in eq. (4.16) to the three-loop estimation given by eq. (5.5). In the comparison, we just keep the dominant contribution from eq. (5.5), arising from the log term which we expand as follows:

$$r = \frac{(m_{\tilde{f}}^2)_{3\text{-loops}}}{(m_{\tilde{f}}^2)_{2\text{-loops}}} \simeq \frac{6}{5\pi} \frac{\alpha_B^2}{\alpha_B - \alpha_k} \frac{S(x, \infty, z)}{S(x, y, z)}. \quad (5.9)$$

This estimate is only valid for $m_{v_k} \gtrsim m_L$. For $\alpha_B^{-1} = 4$, one finds $r \approx 0.16/S(x, y, z)$, showing that the three-loop contributions are rather important, viz. the naive loop factor $\alpha/(4\pi) \approx 0.01$ gets boosted by a factor of 16. The intuitive explanation for this is that the gauge couplings g_{A_k}, g_B are in general stronger than the MSSM gauge coupling g_k .

Throughout the paper we have considered the $\mathcal{G}_B = SU(5)$ invariant case, to preserve the unification already present in the MSSM. However, one might pose the question if unification is really obtainable in this kind of models, as we are forced to work with low-scale mediation. The one-loop beta-function for the gauge group \mathcal{G}_B is IR-free, conformal and asymptotically free for $p \geq 6$, $p = 5$ and $p \leq 4$, respectively. A straightforward analysis shows the following. For $p \leq 5$, perturbative unification can be obtained if we give appropriate masses – of the order of the GUT scale – to parts

Ω	1.55×10^6	7.8×10^5
x, y, z	0.8, 0.1, 1	0.98, 0.1, 1
$M_{\tilde{g}_B}, m_L$	3743, 46823	5429, 27934
$M_{\tilde{g}_1}, M_{\tilde{g}_2}, M_{\tilde{g}_3}$	156, 274, 599	224, 397, 890
$m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}$	2634, 2518, 2507	1572, 1506, 1500
$m_{\tilde{L}}, m_{\tilde{e}}$	843, 410	491, 236
$m_{H_u}^2, m_{H_d}^2$	$736^2, 835^2$	$432^2, 486^2$
$\mu, B\mu$	$956, 1271^2$	$591, 828^2$
$m_{\tilde{g}}$	860	1161
$m_{\tilde{\chi}_0}$	139, 279, 950, 953	200, 385, 587, 603
$m_{\tilde{\chi}_{\pm}}$	279, 954	385, 603
$m_{\tilde{u}_L}, m_{\tilde{d}_L}$	2674, 2675	1677, 1679
$m_{\tilde{u}_R}, m_{\tilde{d}_R}$	2554, 2544	1612, 1607
$m_{\tilde{t}_1}, m_{\tilde{t}_2}$	2356, 2575	1500, 1626
$m_{\tilde{b}_1}, m_{\tilde{b}_2}$	2523, 2573	1593, 1621
$m_{\tilde{e}_R}, m_{\tilde{e}_L}, m_{\tilde{\nu}_e}$	419, 845, 841	248, 504, 498
$m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\tilde{\nu}_\tau}$	407, 843, 838	238, 505, 496
m_{h_0}	119	116
$m_{H_0}, m_{A_0}, m_{H_{\pm}}$	1203, 1203, 1206	730, 730, 734

Table 3. Examples of weak scale spectra in the case of $p = 1$ (two messengers), with $\mu > 0$, $\tan \beta = 20$ and $\alpha_B^{-1} = 4$ at the scale Ω , in the small m_v regime ($m_v \ll \Omega$). The soft masses ($M_{\tilde{g}_B}, m_L$) and ($M_{\tilde{g}_k}, m_{\tilde{Q}, \tilde{u}, \tilde{d}, \tilde{L}, \tilde{e}}$) are given at the scale $m_v = y\Omega$ and $(\mu, B\mu)$ are evaluated at m_v as well.

of the inert mesons in the embedding theory. The magnetic dual theory, however, is not IR-free for $p = 5$ which is why we have considered $p = 6$ in the numerical examples. For $p = 6$, unification is rather marginal. If we take equal copies of the messengers, all at the same scale, we cannot obtain unification due to the coupling g_B running into a Landau pole before the GUT scale. In order to ameliorate this problem, we can keep two messengers as in the numerical examples above and split the remaining five couples by changing the VEV of $\langle Z \rangle \sim \text{diag}(1, z', z', z', z', z')$, with $z' \gg 1$, giving split eigenvalues for the rest of the messengers. This results in a small interval of asymptotically free running and then all the messengers kick in. This way we obtain unification of all the couplings of \mathcal{G}_A and avoid the Landau pole problem for g_B , though we get a somewhat strongly coupled unification, i.e. α_{GUT} of order 1.

Ω	4.1×10^5	3.15×10^5
x, y, z	0.8, 0.1, 1	0.98, 0.1, 1
$M_{\tilde{g}_B}, m_L$	5941, 30672	13156, 29348
$M_{\tilde{g}_1}, M_{\tilde{g}_2}, M_{\tilde{g}_3}$	242, 433, 994	532, 950, 2169
$m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}$	1579, 1516, 1510	1437, 1376, 1370
$m_{\tilde{L}}, m_{\tilde{e}}$	480, 229	451, 218
$m_{H_u}^2, m_{H_d}^2$	$408^2, 474^2$	$408^2, 448^2$
$\mu, B\mu$	$548, 803^2$	$495, 872^2$
$m_{\tilde{g}}$	1254	2407
$m_{\tilde{\chi}_0}$	219, 414, 547, 572	456, 497, 521, 948
$m_{\tilde{\chi}_{\pm}}$	414, 571	489, 948
$m_{\tilde{u}_L}, m_{\tilde{d}_L}$	1690, 1692	1838, 1840
$m_{\tilde{u}_R}, m_{\tilde{d}_R}$	1628, 1623	1778, 1774
$m_{\tilde{t}_1}, m_{\tilde{t}_2}$	1528, 1645	1688, 1801
$m_{\tilde{b}_1}, m_{\tilde{b}_2}$	1611, 1640	1763, 1793
$m_{\tilde{e}_R}, m_{\tilde{e}_L}, m_{\tilde{\nu}_e}$	241, 495, 488	245, 513, 506
$m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\tilde{\nu}_\tau}$	232, 495, 487	237, 513, 505
m_{h_0}	116	116
$m_{H_0}, m_{A_0}, m_{H_{\pm}}$	694, 694, 699	668, 668, 673

Table 4. Examples of weak scale spectra in the case of $p = 6$ couples of messengers, with $\mu > 0$, $\tan \beta = 20$ and $\alpha_B^{-1} = 4$ at the scale Ω , in the small m_v regime ($m_v \ll \Omega$).

5.2.2 $\mathcal{G}_B = U(1) \times SU(2) \times SU(3)$

In this section we discuss the case in which the gauge group \mathcal{G}_B is $U(1) \times SU(2) \times SU(3)$, instead of $SU(5)$, and each of the factors has a *different* gauge coupling g_{B_k} . The charges of the link fields are:

	$U(1)_A \times SU(2)_A \times SU(3)_A$			$U(1)_B \times SU(2)_B \times SU(3)_B$		
L_2	$-\frac{3}{2\sqrt{15}}$	2	1	$\frac{3}{2\sqrt{15}}$	2	1
\tilde{L}_2	$\frac{3}{2\sqrt{15}}$	2	1	$-\frac{3}{2\sqrt{15}}$	2	1
L_3	$\frac{1}{\sqrt{15}}$	1	3	$-\frac{1}{\sqrt{15}}$	1	$\bar{3}$
\tilde{L}_3	$-\frac{1}{\sqrt{15}}$	1	$\bar{3}$	$\frac{1}{\sqrt{15}}$	1	3

At the scale Ω the sfermion masses are taken to be zero, while the gaugino of the gauge group \mathcal{G}_{B_k} gets a soft mass $M_{\tilde{g}, B_k}$, which can be computed from the expression (3.6, 4.14), with α_k replaced by α_{B_k} . The link fields (L_2, L_3) obtain the following

soft masses from gauge mediation:

$$\begin{aligned} m_{L_2}^2 &= 2p \left(\frac{f}{\Omega} \right)^2 S(x, \infty, z) \left(\frac{3}{4} \left(\frac{\alpha_{B_2}}{4\pi} \right)^2 + \frac{3}{20} \left(\frac{\alpha_{B_1}}{4\pi} \right)^2 \right), \\ m_{L_3}^2 &= 2p \left(\frac{f}{\Omega} \right)^2 S(x, \infty, z) \left(\frac{4}{3} \left(\frac{\alpha_{B_3}}{4\pi} \right)^2 + \frac{1}{15} \left(\frac{\alpha_{B_1}}{4\pi} \right)^2 \right). \end{aligned} \quad (5.10)$$

From the scale Ω to m_v , the renormalization group equations are [24]:

$$\begin{aligned} \frac{d M_{\tilde{g}, B_k}}{d(\log \mu)} &= \frac{b_{B_k}}{8\pi^2} g_{B_k}^2 M_{\tilde{g}, B_k}, \quad b_{B_k} = \left(\frac{12}{5}, -2, -3 \right), \\ \frac{d m_{L,2}^2}{d(\log \mu)} &= -\frac{3}{4} \frac{1}{2\pi^2} g_{B_2}^2 M_{\tilde{g}, B_2}^2 - \frac{3}{20} \frac{1}{2\pi^2} g_{B_1}^2 M_{\tilde{g}, B_1}^2, \\ \frac{d m_{L,3}^2}{d(\log \mu)} &= -\frac{4}{3} \frac{1}{2\pi^2} g_{B_3}^2 M_{\tilde{g}, B_3}^2 - \frac{1}{15} \frac{1}{2\pi^2} g_{B_1}^2 M_{\tilde{g}, B_1}^2. \end{aligned} \quad (5.11)$$

At the scale m_v , the following sfermion masses are generated by integrating out the link field and the heavy gaugino:

$$m_{\tilde{f}}^2 = \sum_k \frac{\alpha_k}{4\pi} C_{\tilde{f},k} \left(\frac{2\alpha_k(\alpha_{B_k} - 3\alpha_k)}{\alpha_{B,k}^2} M_{\tilde{g}, B_k}^2 + \frac{\alpha_k}{\alpha_{B_k} - \alpha_k} m_{v_k}^2 \log \left(1 + \frac{2m_{L_k}^2}{m_{v_k}^2} \right) \right),$$

where

$$m_{L_1}^2 = \frac{2m_{L_3}^2 + 3m_{L_2}^2}{5}. \quad (5.12)$$

The Higgs soft masses are given by eq. (5.6) with α_B replaced by $\alpha_{B,3}$. The MSSM gaugino masses are determined from $M_{\tilde{g}, B_k}$:

$$M_{\tilde{g}_k} = \frac{\alpha_k}{\alpha_{B_k}} M_{\tilde{g}, B_k}. \quad (5.13)$$

The trilinear scalar couplings at the scale m_v are now:

$$\begin{aligned} a_t &= \frac{\lambda_t}{4\pi} \left(\frac{16}{3} \frac{\alpha_3^2}{\alpha_{B_3}} M_{\tilde{g}, B_3} + 3 \frac{\alpha_2^2}{\alpha_{B_2}} M_{\tilde{g}, B_2} + \frac{13}{15} \frac{\alpha_1^2}{\alpha_{B_1}} M_{\tilde{g}, B_1} \right), \\ a_b &= \frac{\lambda_b}{4\pi} \left(\frac{16}{3} \frac{\alpha_3^2}{\alpha_{B_3}} M_{\tilde{g}, B_3} + 3 \frac{\alpha_2^2}{\alpha_{B_2}} M_{\tilde{g}, B_2} + \frac{7}{15} \frac{\alpha_1^2}{\alpha_{B_1}} M_{\tilde{g}, B_1} \right), \\ a_\tau &= \frac{\lambda_\tau}{4\pi} \left(3 \frac{\alpha_2^2}{\alpha_{B_2}} M_{\tilde{g}, B_2} + \frac{9}{5} \frac{\alpha_1^2}{\alpha_{B_1}} M_{\tilde{g}, B_1} \right). \end{aligned} \quad (5.14)$$

The Higgs mass gets a contribution which is absent in MSSM, due to the fact that the D-term of the heavy gauge boson does not decouple completely if there is supersymmetry breaking. The usual MSSM D-terms are modified to [9]:

$$V_D = \frac{g_2^2(1 + \Delta_2)}{8} \left| H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d \right|^2 + \frac{3g_1^2(1 + \Delta_1)}{8} \left| H_u^\dagger H_u - H_d^\dagger H_d \right|^2, \quad (5.15)$$

where σ^a are the Pauli matrices and Δ_k are given by:

$$\Delta_k = \frac{\alpha_k}{\alpha_{B_k} - \alpha_k} \frac{2m_{L_k}^2}{m_{v_k}^2 + 2m_{L_k}^2}. \quad (5.16)$$

In the presence of $\Delta_{1,2}$, the usual bound $m_{h_0} < m_Z$ at tree level (which is saturated at large $\tan \beta$) is replaced by [25–28]:

$$m_{h_0}^2 < \tilde{m}^2, \quad \tilde{m}^2 = \frac{\frac{3}{5}g_1^2(1 + \Delta_1) + g_2^2(1 + \Delta_2)}{2} v_h^2, \quad v_h = 174 \text{ GeV}. \quad (5.17)$$

If $\Delta_{1,2}$ is of order one, this contribution is quite useful to ameliorate the little hierarchy problem. For the concrete case considered in tables 1–4, this is negligible because we considered the $SU(5)_B$ invariant coupling $\alpha_B^{-1} = 4$ at the messenger scale, which is large compared to $\alpha_{1,2}$. Choosing for example $\alpha_B^{-1} = 7$ would give an extra contribution to m_{h_0} of up to about 10 GeV. However, for $\alpha_B^{-1} \gtrsim 4$ we will have to give up unification anyway (i.e. α_{GUT} will become strongly coupled) and hence there is no motivation for keeping the $SU(5)$ invariant couplings. So if instead we consider the $U(1) \times SU(2) \times SU(3)$ case and choose e.g. $\alpha_{B_i} \approx \alpha_{A_i}$, the extra contribution to m_{h_0} can be up to about 30 GeV.

We can use SOFTSUSY to compute the mass of all the particles in the spectrum with the exception of the Higgses, which we compute at tree level from the soft masses at the weak scale. The tree-level masses of the Higgses are:

$$m_{A_0}^2 = \frac{m_{H_u}^2 - m_{H_d}^2}{\cos 2\beta} - \tilde{m}^2, \quad m_{H_{\pm}}^2 = \frac{m_{H_u}^2 - m_{H_d}^2}{\cos 2\beta} - \frac{\frac{3}{5}g_1^2(1 + \Delta_1)}{2} v_h^2, \quad (5.18)$$

$$m_{h_0, H_0}^2 = \frac{1}{2} \left(m_{A_0}^2 + \tilde{m}^2 \mp \sqrt{(m_{A_0}^2 - \tilde{m}^2)^2 + 4\tilde{m}^2 m_{A_0}^2 \sin^2 2\beta} \right),$$

where \tilde{m} and v_h are given in eq. (5.17). For h_0 we add the one-loop correction [24]:

$$\Delta_{m_{h_0}^2} = \frac{3}{4\pi^2} (\cos^2 \alpha) \lambda_t^2 m_t^2 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}, \quad (5.19)$$

where α is the Higgs mixing angle, which can be obtained from the tree-level masses as follows:

$$\sin 2\alpha = -\frac{m_{H_0}^2 + m_{h_0}^2}{m_{H_0}^2 - m_{h_0}^2} \sin 2\beta. \quad (5.20)$$

Examples of spectra in the small m_v regime (to allow large $\Delta_{1,2}$) are shown in table 5. The Higgs h_0 can have a mass near 140 GeV, with a stop mass near 900 GeV.

6 Discussion

In this work, we inspected the sparticle spectrum in direct gaugino mediation [3]. The main result is the following. We found a relatively reasonable spectrum already

Ω	3.8×10^5	2.5×10^5
x, z	0.8, 1	0.98, 1
y_1, y_2, y_3	0.010, 0.014, 0.022	0.010, 0.014, 0.022
m_{L_2}, m_{L_3}	4007, 10867	3111, 8639
$M_{\tilde{g}_1}, M_{\tilde{g}_2}, M_{\tilde{g}_3}$	216, 400, 1004	406, 753, 1897
$m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}$	979, 964, 959	663, 651, 648
$m_{\tilde{L}}, m_{\tilde{e}}$	222, 168	155, 117
$m_{H_u}^2, m_{H_d}^2$	$162^2, 218^2$	$124^2, 153^2$
$\mu, B\mu$	$292, 442^2$	$181, 362^2$
$m_{\tilde{g}}$	1132	1932
$m_{\tilde{\chi}_0}$	193, 279, 298, 430	172, 187, 393, 754
$m_{\tilde{\chi}_{\pm}}$	272, 429	180, 754
$m_{\tilde{u}_L}, m_{\tilde{d}_L}$	1086, 1089	908, 911
$m_{\tilde{u}_R}, m_{\tilde{d}_R}$	1070, 1066	891, 889
$m_{\tilde{t}_1}, m_{\tilde{t}_2}$	1035, 1080	867, 909
$m_{\tilde{b}_1}, m_{\tilde{b}_2}$	1057, 1071	883, 897
$m_{\tilde{e}_R}, m_{\tilde{e}_L}, m_{\tilde{\nu}_e}$	178, 244, 230	137, 213, 197
$m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\tilde{\nu}_\tau}$	167, 250, 229	131, 216, 197
m_{h_0}	139	139
$m_{H_0}, m_{A_0}, m_{H_{\pm}}$	488, 487, 497	460, 460, 470
Δ_1, Δ_2	0.88, 0.53	0.95, 0.67

Table 5. Examples of weak scale spectra in the case of $p = 6$ couples of messengers, with $\mu > 0$, $\tan \beta = 20$ and $\alpha_{B_k}^{-1} = \alpha_{A_k}^{-1}$ at the scale m_v , in the small m_v regime ($m_v \ll \Omega$).

in the hybrid case, when the various scales in the problem – the messenger scale Ω , the effective SUSY-breaking scale f/Ω , the R-symmetry breaking scale ω and the Higgsing scale m_v – are comparable. Concretely, from table 2, we see that the gluino mass is near the TeV – above the current LHC limits, while the stop mass is in the 1–2 TeV range. Intriguingly, when the SUSY-breaking scale is sufficiently close to the messenger scale, the NLSP might be a stau, followed by a right-handed selectron with a comparable mass.

From table 4, we see that the superpartner spectrum in the gaugino mediation regime – when the Higgsing scale m_v is much smaller than the messenger scale Ω – is rather similar to the one in the hybrid case. This is due to the large three-loop contributions to the soft scalar masses in this case. It would be interesting to investigate also the intermediate regime, where the two and three-loop contributions to the soft sfermion masses are comparable.

Finally, if we do not require unification – e.g. when both gauge groups in figure 1 are $SU(3) \times SU(2) \times U(1)$ with comparable couplings – and deep in the gaugino

mediation regime, $m_v \ll \Omega$, we find significant contribution to the Higgs potential, which is useful to ameliorate the little hierarchy problem. Concretely, in the second column of table 5, the stop mass is near 900 GeV and the SM Higgs mass is about 140 GeV.

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